Name: ___________________ ***Math 10F&PC H.*** **Date:** __________________

Chapter 3 Factors and Products

3.8 Factoring Special Polynomials

There are a few types of special polynomials that we are going to look at. The first type is ***perfect square trinomials***. All of this type can be written in factored form of a binomial squared.

To explore this type, expand the following:

A **perfect square** has the following form:

$\mathbf{(a + b)^2} =$ OR $\mathbf{(a - b)^2} =$ $(a-b)(a-b)$ FoiL

$(a+b)(a+b)$

mr. marzook $a^2+2ab+b^2$ $a^2-2ab+b^2$

To identify a perfect square trinomial:

1) The first *and* last term must be perfect squares.
2) The middle term must be twice the product of the square roots of the first and last terms. (i.e. 2ab)

Exp. $(2x+5)^2$

$4x^2+20x+25$

Example: $4x^2 + 20x + 25 = (2x + 5)^2$

$(2x)^2+20x+(5)^2 = (2x+5)^2$

Example:

1) Factor each trinomial. Which trinomials are ***perfect squares***?

ax^2+bx+c

a) $y^2 - 10y + 25$

$(y-5)^2$

b) $4t^2 + 4t + 1$

$(2t+1)^2$

c) $16x^2 + 24x + 9$

$(4x)^2+24x+(3)^2$

$(4x+3)^2$

d) $y^2 + 3y + 2$

Factor $(y+2)(y+1)$

We will also encounter **trinomials with 2 variables** and the method we can use to factor this type is decomposition.

e) $36m^2 - 60mn + 25n^2$

$(6m)^2 - 60mn + (5n)^2 = (6m-5n)^2$

A **perfect square** can also exist as a **difference of squares:**

$a^2 - b^2$ $(a - b)(a + b)$ FOIL

$a^2 - 81$

$(a-9)(a+9)$

A difference of squares is a binomial of the form $a^2 - b^2$. We can think of it as a trinomial with a middle term of 0 ($a^2 + 0\,ab - b^2$) The "ab's" subtract each other out so you do not write out the middle (or first-degree) term.

Example:

1) Factor each binomial. Which binomials is difference of squares?

a) $9x^2 - 16y^2$

$(3x+4y)(3x-4y)$

b) $1 - 64y^2$

$(1+8y)(1-8y)$

Examples: Identify each polynomial as a perfect square trinomial, a difference of squares, or neither.

a) $81x^2 - 121p^2$

difference $(9x+11p)(9x-11p)$

b) $169 + z^2$

* Prime

c) $4m^2 - 12m - 9$

* Prime

d) $x^2 + 14xy + 49y^2$

$(x+7y)^2$ perfect square

e) $y^2 - 100$

difference $(y+10)(y-10)$

f) $4x^2 - 20x + 25$

$(2x-5)^2$ perfect square.

\#\# ***Assignment Pages 194-195 Q: 4abef, 5, 6, 8abc, 9, 10aceg, 12ace, 13abf, 14, 15a, 18*** *Note: 9b) is wrong in the back 13f) could also be 2(8c- 3b)(8c + 3b)*

The Binomial Theorem

Here is the expansion of $(x + y)^n$ for $n = 0, 1,\ldots, 5$:

$(x + y)^0 = 1$

$(x + y)^1 = x + y$

$(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Look familiar? The coefficients of each expansion are the entries in Row n of Pascal's Triangle. Thus, the coefficient of each term r of the expansion of $(x + y)^n$ is given by $C(n, r - 1)$. The exponents of x descend, starting with n , and the exponents of y ascend, starting with 0, so the r^{th} term of the expansion of $(x + y)^2$ contains $x^{n-(r-1)}y^{r-1}$.

Example: Write out the expansion of $(2x + 3y)^4$.

1 4 6 4 1

$(2x+3y)^4 = (2x)^4 + 4(2x)^3(3y)^1 + 6(2x)^2(3y)^2 + 4(2x)^1(3y)^3 + (3y)^4$

$= 16x^4 + 4(8x^3 \cdot 3y) + 6(4x^2 \cdot 9y^2) + 4(2x \cdot 27y^3) + 81y^4$

$= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$

1 3 3 1

Example: Write out the expansion of $(5x - y)^3$.

$(5x-y)^3 = (5x)^3 + 3(5x)^2(-y) + 3(5x)(-y)^2 + (-y)^3$

$= 125x^3 + 3(25x^2 \cdot -y) + 3(5x \cdot y^2) - y^3$

$= 125x^3 - 75x^2y + 15xy^2 - y^3$

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Chapter 3 Factors and Products

3.8 Factoring Special Polynomials & Binomial Expansion

Pascal's Triangle

Pascal's Triangle is a triangle in which each row has one more entry than the preceding row, each row begins and ends with "1," and the interior elements are found by adding the adjacent elements in the preceding row. The triangle is symmetrical.

Expand $(x+y)^3$

$1x^3 + 3x^2y^1 + 3x^1y^2 + 1y^3$

1 — row 0

1 1 — row 1

1 2 1 — row 2

1 3 1 — row 3

1 4 6 4 1 — row 4

1 5 10 10 5 1 — row 5

1 6 15 20 15 6 1 — row 6

1 7 21 35 35 21 7 1 — row 7

In Row 6, for example, 15 is the sum of 5 and 10, and 20 is the sum of 10 and 10. Note that the triangle begins with Row 0.

Problem : Write out the expansion of $(x + y)^6$.

$$(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

Problem : Write out the expansion of $(x - 5)^4$.

$$(x-5)^4 = x^4 - 20x^3 + 150x^2 - 500x + 625$$

Problem : Write out the expansion of $(2x + 7y)^3$.

$(2x + 7y)^3 = 8x^3 + 84x^2y + 294xy^2 + 343y^3$

Problem : Write out the expansion of $(1 - x)^7$.

$(1 - x)^7 = 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$

Problem : Write out the expansion of $(x^2 + 3y)^4$.

$(x^2 + 3y)^4 = x^8 + 12x^6y + 54x^4y^2 + 108x^2y^3 + 81y^4$

Use the general formula for Binomial Expansion to expand and simplify each:

a) $(a + 5)^3$

b) $(n - 2)^3$

c) $(2x-7)^4$

d) $(3x - 5y)^3$

e) $(a + 3)^4$

f) $(x - 1)^4$

g) $(mn + 5p)^4$

h) $(6 - n)^4$

i) $(2-3y)^5$

j) $(x + 2)^5$

k) $(a - 1)^6$

l) $(x + 1)^8$